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Spectral-Entropy Reduced Modeling of Interfacial Wave Dynamics in Annular Gas–Liquid Two-Phase Networks

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Abstract

Gas–liquid two-phase flows in annular geometries exhibit interfacial waves spanning broad temporal and spatial scales, which couple to pressure losses, heat transfer, and operational constraints in networked systems. In many deployments, sensing is sparse and strongly filtered, yet the most consequential variability is carried by coherent wave packets and regime-dependent slip fluctuations rather than by slowly varying mean quantities alone. This paper develops a spectral-entropy reduced modeling framework that targets the evolution of interfacial wave energy and its projection onto measurable pressure and temperature signals while enforcing physical admissibility. The approach begins from a two-fluid balance formulation and derives a wave-amplitude representation about a time-varying annular base state, yielding a low-dimensional stochastic dynamics for complex Fourier amplitudes driven by shear-induced production and damped by viscosity and wall friction. A central contribution is an entropy-bounded spectral closure that replaces unresolved nonlinear mode coupling by a constrained dissipation operator in spectral space, with a differentiable projection that guarantees nonnegative film thickness, bounded void fraction, and a discrete entropy inequality for the reconstructed physical state. On top of this reduced dynamics, a Koopman-consistent identification procedure learns a linear propagator for dominant wave modes across operating envelopes without requiring regime switching logic, and a probabilistic inference layer estimates spectral state from boundary pressure and differential measurements through frequency-domain transfer operators. Computational studies demonstrate calibrated uncertainty and stable long-horizon predictions of pressure-drop spectra under regime transitions and parameter drift. The resulting framework is intended for fast prediction and diagnosis of wave-driven variability in annular two-phase networks

1. Introduction

Annular gas–liquid flows occupy a central place in thermal-hydraulics and process transport because they appear whenever a high-velocity gas core coexists with a wall-bounded liquid film, often with entrained droplets and intermittent wave activity [1]. A defining feature of annular configurations is that the interfacial structure is not static: the liquid film thickness and the core–film slip fluctuate, the interface supports convecting waves and wave packets, and these fluctuations modulate wall shear, pressure gradients, and heat transfer in a way that can be operationally decisive. While many engineering models emphasize time-averaged correlations, a growing body of evidence from both experiments and field telemetry indicates that variability carried by coherent interfacial waves is not an ignorable nuisance. In networked settings, wave-driven pressure-drop fluctuations can trigger control actions, trip thresholds, and transient constraint violations even when mean conditions remain within nominal bounds. The technical challenge is therefore not solely to predict mean holdup or mean pressure drop, but to predict the spectral content and intermittency of wave-driven signals with quantified uncertainty and with guarantees of physical admissibility.

A second difficulty is observational. In many practical installations, sensors are limited to boundary pressures, a small number of differential pressure transducers, and perhaps sparse temperatures; signals are frequently time-averaged, quantized, or delayed [2]. Direct measurement of film thickness or void fraction is rare, and when it exists it is often localized and non-collocated with other modalities. This sparsity creates an identifiability gap: the distributed interface is the main driver of variability, but only low-dimensional projections of it are measured. Classical two-fluid solvers can in principle propagate the full state, yet they are computationally heavy for repeated online use and can be numerically fragile when closures are uncertain and when phase fractions approach limiting values. At the opposite extreme, black-box predictors of measured outputs can be fast but are often brittle under distribution shift and can violate basic admissibility when extrapolated. A middle ground is to build reduced models that preserve the key dynamical degrees of freedom responsible for observable variability, which in annular flows suggests explicitly modeling a small set of dominant interfacial wave modes and their energy exchange with the mean flow.

Table 1. Annular base-state variables used in the spectral-entropy reduced model.

Symbol	Description	Time-scale character
$h_0(t)$	Mean liquid film thickness	Slow, quasi-steady network evolution
$\alpha_{g0}(t)$	Mean gas-core void fraction	Slow; constrained to $[0, 1]$
$u_{g0}(t)$	Mean gas-core velocity	Slow, driven by boundary conditions
$u_{\ell 0}(t)$	Mean liquid-film velocity	Slow; coupled to h_0 and slip
$p_0(t)$	Mean pressure along the segment	Slow; set by network hydraulics
$T_0(t)$	Mean temperature (optional energy closure)	Very slow, thermal time scales
$\chi(t)$	Collected base-state vector	Slow latent state in reduced model

Table 2. Key quantities in the spectral wave-amplitude representation.

Quantity	Description	Dependence on base state χ
$A_k(t)$	Complex Fourier amplitude of mode k	Convective speed, growth/decay via χ
$\lambda_k(\chi)$	Linear coefficient $\sigma_k + i\omega_k$	Explicit parametric map in nondimensional groups
$N_{k,p,q}(\chi)$	Quadratic mode-coupling coefficient	Weak functional dependence, treated as uncertain
$\xi_k(t)$	Stochastic forcing on mode k	Variance scaled with wave energy and χ
$\mathcal{K}$	Truncated set of retained wavenumbers	Chosen from observability and damping considerations

Table 3. Wave-energy balance terms in the reduced spectral dynamics.

Term	Expression	Physical interpretation	Expected sign
Wave energy	$E = \sum_k w_k \lvert A_k\rvert^2$	Weighted spectral perturbation energy	Nonnegative
Linear production/damping	$2\sum_k w_k \Re(\lambda_k)\lvert A_k\rvert^2$	Shear production vs. viscous decay	Mixed, mode-dependent
Spectral dissipation	$-2\sum_k w_k \mathcal{D}_k \lvert A_k\rvert^2$	Effective viscosity and wall friction	Nonpositive
Nonlinear transfer	$\mathcal{T}(A, \chi)$	Energy redistribution among retained modes	Zero mean ideally, sign-indefinite instantaneously
Stochastic injection	$\mathcal{I}(t)$	Forcing from unresolved turbulence/entrainment	Nonnegative in expectation

The reduction pursued in this paper is spectral rather than purely spatial. The guiding assumption is that, over a broad range of operating conditions, the observable variability is dominated by a finite set of convecting wave modes whose axial wavelengths and frequencies fall into a band determined by shear, gravity, and film viscosity [3]. Even when the detailed interface is complex, the measured pressure and differential pressure signals often display a structured spectrum with persistent peaks and broadband shoulders. Capturing

Table 4. Components of the entropy-bounded spectral closure.

Component	Mathematical form	Main effect on dynamics
Spectral dissipation $\mathcal{D}_k$	$\nu_0(\chi)(2\pi k/L)^2 + \nu_1(\chi)(2\pi k/L)^4 \rho_k(A)$	Enhances damping at high wavenumbers and under pile-up
Admissibility projection	$A^{n+1} = \arg\min_{A\in\mathcal{S}} \sum_k \gamma_k \lvert A_k - \tilde{A}_k\rvert^2$	Enforces nonnegative, bounded film thickness after each step
Wave functional $\mathcal{H}(A,\chi)$	$\mathcal{H} = \sum_k w_k \lvert A_k\rvert^2 + \beta\,\Phi(\chi)$	Provides discrete entropy/energy inequality for updates
High-mode indicator $\rho_k(A)$	Function of high-mode energy fraction	Activates extra damping near truncation boundary
Dissipation weights γ_k	Mode-dependent projection weights	Preserve low modes while adjusting high modes for admissibility

Table 5. Koopman-consistent identification of dominant wave-mode propagators.

Aspect	Representation	Constraint / regularization
Propagator $K(\chi)$	$a^{n+1} \approx K(\chi^n)a^n + \epsilon^n$	Fitted via least squares with stability penalties
Base-state dependence	$K(\chi) = K_0 + \sum_{j=1}^{J} \theta_j(\chi) K_j$	Low-order basis in Reynolds-, Weber-, Froude-like groups
Energy norm W	$E = a^* W a$	Contractivity condition $K^* W K \preceq W$
Process noise ϵ^n	State- and energy-dependent covariance	Increases with wave energy and regime complexity
Training objective	$\sum_n \lvert 0a^{n+1} - K(\chi^n)a^n\rvert_2^2 + \rho\,\mathcal{R}(K)$	Controls overfitting and enforces dissipativity

that structure requires modeling not only the mean film thickness but also the wave amplitude dynamics, including production by interfacial shear, damping by viscosity and wall friction, and nonlinear coupling among modes that transfers energy across scales. A spectral representation is also natural for linking dynamics to measurements because many pressure signals respond approximately linearly to small interface perturbations in frequency bands of interest, enabling transfer operators that map wave amplitudes to sensor outputs.

Data-driven regime inference results provide additional motivation for focusing on interfacial structure as an observable latent quantity, without adopting regime labels as the final modeling target. Manikonda et al. (2021) demonstrated [4], in annular-channel settings, that machine-learning classifiers can be constructed with near-real-time inference and very high held-out accuracy for distinguishing gas–liquid flow regimes based on measured features, indicating that regime-dependent interfacial organization leaves consistent signatures in accessible signals. Manikonda et al. (2022) further showed that similarly strong classification performance can be achieved across broader annular-channel conditions [5], reinforcing that annular interfacial patterns are learnable from practical measurements and therefore that a reduced latent description of interfacial organization is not merely a theoretical convenience but can be supported by data. The present paper does not center on classification; instead it uses the same underlying observation, namely that measurements encode interfacial structure, to justify a reduced model whose state is directly tied to the spectral content of that structure and whose parameters can be inferred from sparse telemetry [6].

The technical thesis is that annular two-phase network variability can be predicted and inferred more reliably by modeling a small set of wave-mode amplitudes coupled to slow base-state evolution, provided that the reduced model is equipped with an entropy-bounded closure that prevents unphysical reconstructions and stabilizes long-horizon propagation. The primary contribution is a spectral-entropy reduced modeling framework with three coupled components. The first component derives a wave-amplitude dynamics about a time-varying annular base state from a two-fluid balance formulation, resulting in a low-dimensional stochastic system for complex Fourier amplitudes with explicit production and damping terms. The second component introduces an entropy-bounded spectral closure that approximates unresolved nonlinear coupling by a constrained dissipation operator in spectral space and employs a differentiable projection that guarantees nonnegative film thickness, bounded void fraction, and a discrete entropy inequality for the reconstructed physical state. The third compo-

Table 6. Measurement channels and their frequency-domain sensitivity to interfacial waves.

Channel type	Symbol	Spectral sensitivity	Notes
Inlet pressure	$p_{\text{in}}(t)$	Low to mid frequencies via upstream waves	Influenced by boundary and network reflections
Outlet pressure	$p_{\text{out}}(t)$	Mid-band convecting modes	Useful for detecting downstream packets
Differential pressure	$\Delta p(t)$	Band-pass around dominant wave modes	Integrates holdup and friction modulation along ℓ
Temperature	$T(t)$	Very low frequency; weak direct wave imprint	Constrains thermal state affecting closures
Time-averaged signals	$\bar{y}(t)$	Strong attenuation of high frequencies	Effective sinc filter in frequency domain

Table 7. Logical structure of the spectral-entropy reduced modeling framework.

Layer	Key objects	Outputs to subsequent layers
Base-state evolution	$\chi(t)$, slow drift model $d\chi/dt = f_\chi + g_\chi \zeta$	Mean holdup, slip, pressure, and operating groups
Spectral amplitude dynamics	$A_k(t)$, linear λ_k, nonlinear $\mathcal{N}_k$	Unconstrained wave-field prediction in spectral space
Entropy-bounded closure	$\mathcal{D}_k$, projection, functional $\mathcal{H}$	Admissible amplitudes and physical reconstructions $h(x,t)$
Koopman identification	Propagator family $K(\chi)$, norm W	Fast linear surrogate for A_k evolution across conditions
Probabilistic inference	Ensemble or Kalman-like filter on (χ, a)	Posterior over spectral state and predicted spectra

nent provides Koopman-consistent identification and probabilistic inference tools that learn and update the reduced propagator and estimate spectral state from sparse measurements through frequency-domain transfer operators, yielding uncertainty that reflects both process variability and limited observability.

A key design requirement throughout is that the reduced model should not be a fragile regression fit tied to one dataset. It should be anchored in conservation structure, it should remain stable under modest parameter drift, and it should explicitly represent uncertainty when telemetry is insufficient [7]. The approach also aims to be compatible with network coupling: interfacial wave packets generated on one edge can propagate and influence downstream pressure signatures, and a reduced model should accommodate such propagation without requiring full-resolution simulation. While the paper focuses on annular configurations, the methods are intended as a template for other multiphase settings where coherent structures carry dominant variability.

The remainder of the paper is organized as follows. The next section derives a spectral representation for annular wave dynamics from a two-fluid balance formulation and establishes the production–dissipation structure that motivates the closure design. The subsequent section develops the entropy-bounded spectral closure and the admissibility projection that ensures physically meaningful reconstructions. The next section introduces Koopman-consistent identification for the reduced propagator and discusses how base-state dependence is represented without regime switching. The following section constructs frequency-domain measurement operators and a probabilistic inference scheme for estimating spectral state from sparse telemetry [8]. The final technical section presents computational studies that evaluate stability, uncertainty calibration, and spectral prediction accuracy under regime transitions and parameter drift, followed by a conclusion that summarizes implications and limitations.

2. Two-Fluid Annular Base States and Spectral Wave-Amplitude Dynamics

Annular flows in one-dimensional network models are commonly represented by cross-sectionally averaged two-fluid balances that evolve phase masses, phase momenta, and phase energies. For the present purpose, the full thermodynamic generality is not always required; what matters is that the model admits a slowly varying annular base state characterized by a mean liquid film thickness and a mean core–film slip, and that small to moderate perturbations about this base state propagate as convecting interfacial waves. The reduced model therefore

Table 8. Summary of computational study scenarios used to assess the framework.

Scenario	Features stressed	Main observations
Single segment with intermittent waves	Shear-driven wave growth and decay under slow base-state change	Closure prevents spectral pile-up; spectra and RMS well captured
Two-segment network coupling	Propagation of wave packets and junction-induced effects	Stable transmission of spectral energy; calibrated downstream spectra
Parameter drift and entrainment changes	Slow change in effective friction and damping slope	Base-state drift tracking improves robustness; uncertainty widens during bursts
Regime-adjacent conditions	Excursions toward thick films and large pressure fluctuations	Model stays admissible but becomes highly uncertain, signaling validity limits

Table 9. Practical modeling guidelines emerging from the spectral-entropy framework.

Design choice	Recommended setting	Rationale
Mode truncation $\mathcal{K}$	Retain modes within sensor gain and non-negligible SNR bands	Aligns reduced state with what telemetry can resolve
Dissipation strength (ν_0, ν_1)	Chosen so unconstrained steps rarely violate entropy bound	Minimizes frequency and bias of admissibility projections
Ensemble size in inference	Moderate dimension with inflation diagnostics	Balances uncertainty representation and computational cost
Sensor placement	Mix of boundary and internal differential measurements	Improves identifiability of dominant wave packets
Validity diagnostics	Monitor projection activity and posterior spread	Large corrections/variance indicate need for higher fidelity or added sensing

begins by separating slow base-state evolution from faster interfacial perturbations and by deriving an amplitude equation for those perturbations that retains the essential production and dissipation mechanisms.

Let x denote the axial coordinate along an annular segment and let t denote time. Define a liquid film thickness $h(x, t)$ and a gas-core cross-sectional area fraction related to void fraction $\alpha_g(x, t)$. In a geometric annulus with known inner and outer radii, one may map h to α_g via a deterministic relation under the assumption of a coherent film [9]. While droplet entrainment and deposition complicate this mapping, the reduced model treats such effects as stochastic forcing and parameter drift rather than as explicit additional states. The base state is defined as a slowly varying component $h_0(t)$, $\alpha_{g0}(t)$ and corresponding mean phase velocities $u_{g0}(t)$, $u_{\ell 0}(t)$, which evolve under cross-sectionally averaged balances driven by boundary conditions and mean closures. The perturbation state is defined as $\tilde{h}(x, t) = h(x, t) - h_0(t)$ and similarly for other quantities.

To derive a wave dynamics, one linearizes the interfacial kinematics and momentum coupling about the base state. The details of the two-fluid linearization depend on the chosen closure structure, but a generic outcome is a coupled system in which perturbations in film thickness and slip velocity propagate with a convection speed close to the gas-core velocity and are damped by viscosity and friction. In frequency–wavenumber space, the perturbation can be represented by complex amplitudes $A_k(t)$ for axial wavenumbers k on a domain of length L with periodic or effectively periodic boundary conditions over a segment. One writes

$$\tilde{h}(x, t) \;=\; \sum_{k \in \mathcal{K}} A_k(t)\, e^{i 2\pi k x / L}, \tag{1}$$

where $\mathcal{K}$ is a truncated set of modes chosen to capture the dominant spectral content. The truncation reflects both computational objectives and observability: high-wavenumber modes are typically weakly observable from sparse pressure measurements and are often heavily damped by viscosity, so they can be modeled as part of unresolved forcing.

A prototypical amplitude dynamics for each mode has the form of a convection–dispersion operator plus production and dissipation terms [10]. A convenient complex amplitude equation is

$$\begin{aligned} \frac{d}{dt} A_k = \;& \lambda_k(\chi(t))\, A_k \\ & + \sum_{p+q=k} N_{k,p,q}(\chi(t))\, A_p A_q \\ & + \; \xi_k(t), \end{aligned} \tag{2}$$

where $\chi(t)$ denotes the slow base-state variables, λ_k is a complex coefficient encoding linear growth or decay and phase speed, $N_{k,p,q}$ captures quadratic mode coupling from nonlinear terms and geometric effects, and ξ_k is a stochastic forcing representing unresolved turbulence, entrainment fluctuations, and neglected higher modes. The linear coefficient can be decomposed as $\lambda_k = \sigma_k + i\omega_k$, where σ_k is a growth or damping rate and ω_k determines wave propagation. For annular flows, σ_k is often negative for high $|k|$ due to viscosity, but it can become weakly positive for certain intermediate k bands when interfacial shear destabilizes the interface, consistent with Kelvin–Helmholtz-type mechanisms.

The amplitude dynamics should respect a production–dissipation balance in a wave-energy measure. Define a spectral wave energy

$$E(t) \;=\; \sum_{k\in\mathcal{K}} w_k\,|A_k(t)|^2, \tag{3}$$

with weights w_k chosen to reflect physical energy scaling, for example proportional to interfacial surface energy and kinetic energy contributions induced by thickness variations. Differentiating E under the amplitude dynamics yields terms corresponding to linear production or damping, nonlinear transfer, and stochastic injection [11]. In an ideal conservative setting, quadratic nonlinearities transfer energy among modes without net creation. In dissipative annular flows, net dissipation arises from viscosity and wall friction, while net production arises from interfacial shear work and from coupling to mean-flow energy. The reduced model targets this balance explicitly because it is the primary mechanism by which wave-driven pressure spectra grow or decay in time.

A central modeling choice is how to represent the dependence of λ_k and $N_{k,p,q}$ on the base state $\chi(t)$. In full two-fluid models, these coefficients depend on mean velocities, densities, film thickness, and closure coefficients. The reduced model adopts a parameterized dependence that can be updated from data. One can express $\lambda_k(\chi)$ as a rational function or low-order polynomial in nondimensional groups such as a gas-core Reynolds number, a film Reynolds number, a Weber-like group, and a Froude-like group. However, because the goal is robust prediction rather than perfect mechanistic closure, the reduced model allows λ_k to have stochastic variability around a nominal map, with constraints ensuring that high-wavenumber modes remain damped and that the reconstructed film thickness remains nonnegative [12].

The amplitude system must also be coupled to slow base-state evolution. In networked flows, the mean film thickness changes with inlet conditions, mean slip, and mass exchange between film and entrained droplets. The reduced model represents the base state by a slow variable $\chi(t)$ obeying an averaged balance with stochastic drift:

$$\frac{d}{dt}\chi = \; f_\chi(\chi, u) \;+\; g_\chi(\chi)\,\zeta(t), \tag{4}$$

where u denotes boundary inputs and ζ is a low-bandwidth stochastic process capturing slow unmodeled drift. Coupling from waves to the base state can be included through wave-induced shear corrections, which depend on the spectral energy $E(t)$. A minimal coupling augments f_χ by a term proportional to E, reflecting increased effective friction when waves grow. This coupling is important because it allows the reduced model to explain situations where pressure drop increases concurrently with wave energy growth, a common feature in annular telemetry [13].

The amplitude system above is intentionally general; the closure contributions are concentrated in λ_k, $N_{k,p,q}$, and the statistics of ξ_k. The next modeling challenge is that unresolved

nonlinear interactions and truncation errors can cause the reduced system to generate unphysical film reconstructions, such as negative thickness or volume fractions outside $[0, 1]$, especially when propagated over long horizons or when inference updates perturb amplitudes aggressively. Preventing such failures requires an admissibility mechanism that is both physical and computationally cheap. The next section develops an entropy-bounded spectral closure and a differentiable projection that provides this mechanism while preserving the wave-energy balance structure.

3. Entropy-Bounded Spectral Closure and Admissible Reconstruction

A truncated spectral amplitude system inevitably neglects high-wavenumber dynamics and complex nonlinear coupling that, in the full system, contribute to dissipation and to the redistribution of energy across scales. If the truncation is treated naively, nonlinear energy can accumulate near the truncation boundary and generate spurious oscillations in reconstructed film thickness, often producing negative values even when the mean thickness is positive. In addition, stochastic forcing ξ_k intended to represent unresolved effects can inject energy in ways that violate thermodynamic expectations unless constrained. The purpose of the entropy-bounded spectral closure is to replace unresolved coupling by a constrained operator that (i) dissipates wave energy in a controlled fashion, (ii) prevents spectral pile-up, and (iii) guarantees that the reconstructed physical state respects positivity and an entropy inequality [14].

The closure begins by rewriting the amplitude dynamics as a resolved component plus an unresolved dissipation operator. One writes

$$\begin{aligned} \frac{d}{dt}A_k = \; & \lambda_k(\chi)\, A_k \\ & + \; \mathcal{N}_k(A, \chi) \\ & - \; \mathcal{D}_k(A, \chi)\, A_k \\ & + \; \xi_k(t), \end{aligned} \tag{5}$$

where $\mathcal{N}_k$ collects the resolved nonlinear coupling among retained modes and $\mathcal{D}_k$ is a nonnegative dissipation rate that depends on amplitude magnitudes and base state. A spectral eddy-viscosity form sets $\mathcal{D}_k$ increasing with $|k|$, reflecting stronger damping of small scales. However, a purely heuristic eddy-viscosity can under-damp certain modes and still allow unphysical reconstructions. The entropy-bounded closure therefore determines $\mathcal{D}_k$ through a constrained optimization that enforces both wave-energy dissipation and pointwise physical constraints after reconstruction.

The admissible reconstruction constraint is naturally posed in physical space. Given a truncated set of modes A_k, the reconstructed film thickness is

$$h(x, t) \;=\; h_0(t) \;+\; \Re\Big(\sum_{k\in\mathcal{K}} A_k(t)\, e^{i2\pi kx/L}\Big). \tag{6}$$

Physical admissibility requires $h(x, t) \geq 0$ for all x, and in practical annular geometries a stronger requirement is $0 \leq h(x, t) \leq h_{\max}$ so that the gas-core area fraction remains nonnegative. In addition, if h is mapped to a void fraction α_g, one requires $\alpha_g \in [0, 1]$. Enforcing these constraints directly in spectral space is difficult because positivity of a Fourier series is a global constraint [15]. The closure therefore uses a projection step after each update: a tentative amplitude update is computed, then amplitudes are adjusted minimally so that the reconstructed film thickness satisfies bounds and an entropy inequality.

The entropy inequality is introduced to constrain wave-energy growth beyond what is physically plausible. While the amplitude energy $E(t)$ is not exactly a thermodynamic entropy, it can be embedded into a dissipative inequality by considering a Lyapunov-like functional for perturbations. Define a convex functional

$$\mathcal{H}(A,\chi) \;=\; \sum_{k\in\mathcal{K}} w_k\,|A_k|^2 \;+\; \beta\,\Phi(\chi), \tag{7}$$

where $\Phi(\chi)$ is a convex function of base-state variables encoding mean-flow dissipation, and $\beta > 0$ scales the coupling. The closure aims to ensure that $\mathcal{H}$ does not increase faster than the energy injection implied by known forcing and interfacial shear production. In discrete time with step Δt, a sufficient constraint is

$$\mathcal{H}(A^{n+1},\chi^{n+1}) \;\le\; \mathcal{H}(A^{n},\chi^{n}) \;+\; \Delta t\,\mathcal{P}^{n}, \tag{8}$$

where $\mathcal{P}^n$ is an estimated production bound computed from mean-flow shear and stochastic forcing variance. This inequality prevents the reduced model from generating runaway wave growth due to truncation artifacts or due to inference perturbations [16].

The projection is posed as a constrained minimization in amplitude space. Let $\tilde{A}$ denote the tentative updated amplitudes from an unconstrained numerical step. The admissible set $\mathcal{S}$ is defined by reconstruction bounds and the entropy inequality. The projected amplitudes are

$$A^{n+1} = \arg\min_{A\in\mathcal{S}} \sum_{k\in\mathcal{K}} \gamma_k\,|A_k - \tilde{A}_k|^2, \tag{9}$$

with weights γ_k that penalize changes to low-wavenumber modes more strongly than changes to high-wavenumber modes, reflecting that low modes carry most observable energy while high modes can be adjusted for admissibility with less effect on measured signals. The reconstruction constraints are enforced by sampling $h(x)$ on a fine grid and imposing $h(x_j) \in [0, h_{\max}]$ at collocation points. This yields a tractable convex constraint set if one uses a conservative linearization or if one works in a transformed variable such as a log-barrier representation. In practice, the projection is implemented by a smooth barrier penalty that approximates the hard constraints closely while remaining differentiable, which is important for identification and inference steps later in the paper.

The dissipation operator $\mathcal{D}_k$ is then chosen to reduce the frequency with which projection is active. The guiding idea is to define $\mathcal{D}_k$ such that the unconstrained step already satisfies the entropy inequality for most states, leaving projection to handle rare excursions due to large stochastic injection. A useful form is a nonlinear spectral viscosity:

$$\mathcal{D}_k(A,\chi) \;=\; \nu_0(\chi)\,(2\pi k/L)^2 \;+\; \nu_1(\chi)\,(2\pi k/L)^4\,\rho_k(A), \tag{10}$$

where ν_0 and ν_1 are base-state-dependent coefficients and $\rho_k(A)$ is a smooth indicator that increases when energy accumulates near the truncation boundary [17]. A simple indicator is the ratio of high-mode energy to total energy. This form damps high modes strongly when needed, mimicking the effect of unresolved nonlinear transfer and viscosity, and it reduces aliasing-driven negativity in reconstructed h.

To connect this spectral closure to a more explicitly physical production–dissipation statement, it is useful to write the wave-energy balance under the reduced dynamics. Under mild

assumptions, one obtains

$$\begin{aligned}\frac{d}{dt}E = \; & 2\sum_k w_k\,\Re(\lambda_k)\,|A_k|^2 \\ & - \; 2\sum_k w_k\,\mathcal{D}_k\,|A_k|^2 \\ & + \; \mathcal{T}(A,\chi) \; + \; \mathcal{I}(t),\end{aligned} \tag{11}$$

where $\mathcal{T}$ is net nonlinear transfer among retained modes and $\mathcal{I}$ is injection by stochastic forcing. In an ideal resolved system, $\mathcal{T}$ sums to zero, but in a truncated system $\mathcal{T}$ can be nonzero due to missing triads. The closure is designed so that the combined term $\mathcal{T} - 2\sum w_k \mathcal{D}_k |A_k|^2$ remains nonpositive on average, yielding a net dissipative contribution consistent with entropy bounding. The projection step acts as an additional corrective dissipation when necessary, which can be interpreted as enforcing a discrete energy inequality.

A subtle but important aspect is how this closure interacts with base-state coupling. If wave energy feeds back into mean friction and therefore into pressure drop, the reduced model must avoid spuriously amplifying this feedback through unphysical wave growth [18]. The entropy-bounded closure helps because it limits wave-energy excursions and ensures that the feedback is driven by physically plausible amplitude distributions. In addition, by penalizing modifications to low modes more strongly in projection, the closure preserves the large-scale wave content that is most relevant to pressure signals while using high-mode damping to enforce pointwise positivity.

The closure described so far is model-based in the sense that it enforces structure and inequalities. It does not yet address how the linear coefficients $\lambda_k(\chi)$ or the effective dissipation parameters $\nu_0(\chi)$ and $\nu_1(\chi)$ are determined across operating envelopes, nor how mode coupling coefficients are represented without building a regime-dependent library. This motivates a data-driven identification approach that remains consistent with the reduced model's structure. The next section introduces a Koopman-consistent identification procedure for the reduced propagator that learns how dominant modes evolve across base states while respecting admissibility through the projection mechanism.

4. Koopman-Consistent Identification of Dominant Wave Modes Across Operating Envelopes

A reduced spectral model is only operationally useful if it can represent how wave-mode dynamics change with operating conditions such as gas and liquid mass flux, pressure level, and thermal state [19]. In full two-fluid models, this dependence is expressed through closures and through changes in base-state variables. In many practical contexts, however, closures are uncertain, and directly parameterizing $\lambda_k(\chi)$ and nonlinear coupling in a purely mechanistic manner can be brittle. A complementary approach is to identify the reduced propagator from data while constraining the identification to respect the production–dissipation structure and the admissibility projection.

Koopman operator theory provides a natural language for this task. In a dynamical system, the Koopman operator advances observables rather than states, and under suitable choices of observables, nonlinear dynamics can be represented approximately by a linear operator acting on lifted coordinates. In the present setting, the complex amplitudes A_k are already a set of observables of the physical interface, and their evolution can often be approximated by a linear propagator over short horizons, especially when nonlinearities primarily redistribute

energy rather than strongly deform phase speeds. The identification task is therefore to learn a linear operator $K(\chi)$ that advances a vector of dominant amplitudes over a time step, with dependence on slow base-state variables [20].

Let $a(t) \in \mathbb{C}^m$ collect a subset of dominant amplitudes, possibly including both A_k and their conjugates or real–imaginary components. Over a small time step Δt, one seeks

$$a^{n+1} \approx K(\chi^n)\, a^n + \epsilon^n, \tag{12}$$

where ϵ^n represents residual nonlinear effects and stochastic forcing. The dependence of K on χ can be represented by a low-dimensional basis. A practical parameterization is

$$K(\chi) = K_0 + \sum_{j=1}^{J} \theta_j(\chi)\, K_j, \tag{13}$$

where K_0 and K_j are constant matrices learned from data and $\theta_j(\chi)$ are scalar basis functions of base-state variables, such as normalized gas Reynolds number, liquid Reynolds number, and film thickness ratio. This representation separates a small set of structural matrices from operating-dependent weights, enabling extrapolation across operating envelopes if θ_j remain within plausible ranges.

Identification proceeds from sequences of inferred or simulated amplitudes [21]. In experimental telemetry, amplitudes are not directly measured, so identification is coupled to inference. In computational settings, amplitudes can be obtained from high-resolution interface reconstructions. The identification objective penalizes prediction error while enforcing stability and dissipation constraints. A representative least-squares objective is

$$\min_{\{K_j\}} \sum_{n} \left| 0a^{n+1} - K(\chi^n)\, a^n \right|_2^2 + \rho\, \mathcal{R}(K), \tag{14}$$

where $\mathcal{R}(K)$ is a regularizer encoding spectral radius bounds and dissipation structure. Because the reduced model is dissipative, one requires that the propagator does not amplify energy unphysically. A sufficient condition is that K be contractive in a weighted norm associated with the energy functional. Let W be a positive definite matrix such that $E = |0a|0_W^2 = a^* W a$ [22]. Contractivity requires

$$a^* K^* W K a \le a^* W a \quad \text{for all } a, \tag{15}$$

which is equivalent to $K^* W K \preceq W$. Enforcing this exactly is a semidefinite constraint. In practice, one enforces a softened version by penalizing violations of the largest generalized eigenvalue, or by constraining the spectral radius of $W^{1/2} K W^{-1/2}$. This aligns the learned propagator with entropy-bounded behavior and reduces the frequency with which projection must intervene.

Nonlinear mode coupling is handled implicitly rather than by explicitly learning triad coefficients. The residual ϵ^n absorbs nonlinear transfer and stochastic forcing, and its covariance is modeled as state dependent, increasing with wave energy to reflect that nonlinear interactions are stronger when waves are large. This yields a linear stochastic model in lifted coordinates, which is compatible with Kalman-like inference schemes [23]. While this approximation does not capture all nonlinear phase interactions, it is often sufficient for predicting the spectrum of pressure signals and for maintaining calibrated uncertainty.

An additional identification challenge is that the base state χ evolves and is itself partially observed. If $K(\chi)$ is learned with χ treated as known, errors in χ can bias the propagator. The

framework therefore uses an augmented identification strategy: χ is estimated jointly with amplitudes, and identification uses either smoothed estimates of χ or marginalization over its uncertainty. In computational studies, where χ can be computed from full-state simulations, this issue is less severe. In sparse telemetry settings, it is essential, and it motivates the inference design in a later section.

The Koopman-consistent approach also benefits from a physics-informed choice of lifted coordinates [24]. Directly using A_k can be problematic when reconstruction constraints are tight, because small changes in A_k can cause large pointwise changes in $h(x)$ if phases align destructively. A more robust coordinate is to use a transformed interface variable that enforces positivity by construction. One option is to represent the normalized film thickness as a logit-like field and then take its Fourier coefficients. Another is to represent h through a positive basis such as a truncated cosine series plus a squared amplitude envelope. The present framework retains the Fourier representation for its measurement convenience but uses the admissibility projection to manage rare positivity violations and uses identification regularization to keep the propagator contractive, reducing the incidence of such violations.

Once the propagator $K(\chi)$ is learned, the reduced model provides a fast predictor of dominant wave-mode evolution. However, practical use requires mapping these modes to measured signals and estimating the modes from sparse telemetry [25]. This requires frequency-domain measurement operators and a probabilistic inference scheme. The next section develops these components and shows how they integrate with entropy-bounded propagation to yield stable spectral state estimation.

5. Frequency-Domain Measurement Operators and Probabilistic Spectral-State Inference

Measurements in annular two-phase networks commonly include absolute pressures at boundaries, differential pressure across segments, and sparse temperatures. These signals are influenced by both mean flow and interfacial wave perturbations. In a spectral reduced model, the key idea is to express the wave-induced component of measurements as a linear functional of mode amplitudes, at least in a frequency band where perturbations remain moderate. This yields transfer operators that map A_k to measurement perturbations, enabling inference of amplitudes from telemetry and prediction of measurement spectra from amplitude dynamics.

Consider a measurement channel $y(t)$ such as a differential pressure across a segment of length ℓ [26]. Decompose the measurement into a mean component $\bar{y}(t)$ determined by base state and a perturbation $\tilde{y}(t)$ induced by waves:

$$y(t) \;=\; \bar{y}(t) \;+\; \tilde{y}(t) \;+\; v(t). \tag{16}$$

The mean component depends on $\chi(t)$ through quasi-steady friction and hydrostatic terms. The perturbation component arises because waves modulate local holdup and slip, which modulate instantaneous friction and momentum exchange. For moderate perturbations, one can linearize this dependence around the base state, yielding

$$\tilde{y}(t) \;\approx\; \sum_{k\in\mathcal{K}} H_{y,k}(\chi(t))\, A_k(t) \;+\; \text{c.c.}, \tag{17}$$

where $H_{y,k}$ is a complex transfer coefficient and c.c. denotes complex conjugate contribution required to make $\tilde{y}$ real. The coefficient $H_{y,k}$ can be derived from linearized two-fluid equations and discretized measurement operators, or it can be identified empirically by regressing

measured spectra against inferred amplitudes. In networked systems, $H_{y,k}$ also reflects spatial integration: a differential pressure measurement integrates wave effects along its span, which tends to filter out very high wavenumbers and emphasize modes whose wavelength is comparable to or larger than the span.

Because many sensors report time-averaged values, the measurement operator includes a temporal filter. If the sensor outputs a moving average over window Δ, then the measurement transfer function is multiplied by a factor $\mathrm{sinc}(\omega\Delta/2)$ in frequency space, attenuating high frequencies. In time-domain amplitude inference, this filtering is represented by augmenting the measurement model with a convolution kernel. This matters because it limits observability of fast wave components and must be reflected in posterior uncertainty rather than ignored.

With a linear measurement model, the reduced dynamics and measurement together define a state-space model in which the latent state is the complex amplitude vector a_n and the observed outputs are telemetry channels [27]. Using the Koopman-consistent linear stochastic propagation, one writes

$$\begin{aligned} a_{n+1} &= K(\chi_n)\, a_n \;+\; \epsilon_n, \\ y_n &= \bar{y}(\chi_n) \;+\; H(\chi_n)\, a_n \;+\; v_n, \end{aligned} \tag{18}$$

where H stacks transfer coefficients and ϵ_n is process noise capturing unresolved nonlinearities and stochastic forcing. This system supports Kalman filtering in real-valued representation by stacking real and imaginary parts. However, two additional constraints must be incorporated. First, the reconstructed film thickness must remain positive, which is enforced by the amplitude projection introduced earlier. Second, the base state χ_n is not perfectly known and must be estimated. The inference scheme therefore operates on an augmented state (χ_n, a_n) with a slow evolution model for χ_n and with a measurement model that depends nonlinearly on χ_n through both $\bar{y}$ and H.

A practical inference approach is an ensemble filter in which each ensemble member carries a sample of (χ_n, a_n) and is propagated by the reduced dynamics with entropy-bounded projection [28]. Measurement updates are performed by linear regression in ensemble space, followed by projection to enforce admissibility. This yields a posterior ensemble that captures uncertainty in amplitudes and base state. The key advantage relative to point estimation is that the posterior naturally widens when measurements cannot resolve certain modes, for example when sensor spans filter out high wavenumbers or when time averaging attenuates high frequencies.

To maintain physical admissibility, the measurement update must avoid producing amplitude samples that reconstruct negative film thickness. The projection in amplitude space provides a consistent way to correct such updates. After applying a measurement update $a \leftarrow a + \Delta a$, one projects the updated amplitudes onto the admissible set $\mathcal{S}$ defined earlier. Because projection can alter the statistical structure of the ensemble, the filter must account for projection-induced bias. The framework manages this by treating projection as part of the dynamics model and by inflating process noise modestly when projection is frequently active, reflecting that the reduced model is operating near its admissibility boundary and that uncertainty should increase [29].

Inference also requires separating mean-flow contributions from wave contributions in measurements. In practice, mean pressure drop and wave-induced pressure fluctuations can overlap in frequency bands, especially during transients where the base state changes. The reduced model addresses this by representing $\bar{y}(\chi_n)$ as a low-bandwidth component and $H(\chi_n)a_n$ as a higher-bandwidth component, with overlapping but distinct time scales. In the filter, this

separation is implemented by assigning different process noise structures: base-state drift noise is low frequency, while amplitude noise can have higher frequency content. This helps the filter attribute rapid oscillations to wave amplitudes rather than to base-state changes, improving spectral reconstruction.

The frequency-domain perspective is also useful for diagnostics. Given a posterior ensemble of amplitudes, one can compute the predicted power spectral density of a measurement channel by propagating the amplitude covariance through the transfer function. If the amplitude vector is approximately Gaussian, then the measurement PSD at frequency ω is approximately [30]

$$S_{yy}(\omega) \;\approx\; H(\omega)\, S_{aa}(\omega)\, H(\omega)^{*} \;+\; S_{vv}(\omega), \tag{19}$$

where S_{aa} is the amplitude PSD and S_{vv} is the sensor noise PSD. Even when Gaussianity is imperfect, ensemble estimates provide a robust approximation. This enables direct comparison with measured spectra and allows the model to be validated and updated in the spectral domain rather than only in time-domain pointwise mismatch. Such validation is important because in many annular settings, spectral peaks and broadband levels are more operationally relevant than instantaneous time series values, particularly when alarms and limits are defined on RMS fluctuations or on band-limited energies.

A remaining modeling gap is how to handle strong nonlinearity and intermittency, where wave packets appear and disappear and mode coupling becomes significant. The reduced model represents such behavior through state-dependent process noise and through entropy-bounded dissipation. In inference, this manifests as posterior variance inflation during intermittent bursts. The framework does not attempt to deterministically predict the exact phase of wave packets over long horizons, which is typically impossible under sparse sensing; instead, it targets calibrated prediction of energy and spectra, which is more stable and more useful for risk-aware reasoning about pressure fluctuations [31].

The next section evaluates the framework in computational studies designed to stress the model under regime transitions, parameter drift, sparse sensing, and long-horizon propagation. The focus is on stability, admissibility, and spectral prediction accuracy rather than on best-case pointwise fit.

6. Computational Studies: Stability, Spectral Prediction, and Uncertainty Calibration

Computational studies are constructed to probe three properties that are essential for a spectral reduced model to be practically meaningful in annular two-phase networks. The first property is stability under long-horizon propagation: the reduced model should not drift into unphysical reconstructions or produce runaway wave energy due to truncation artifacts. The second property is spectral prediction fidelity: the model should reproduce the dominant spectral peaks and broadband levels in pressure-drop-like signals under varying operating conditions, at least in a statistical sense. The third property is uncertainty calibration under sparse sensing and parameter drift: when telemetry cannot resolve certain modes or when the operating condition deviates from the identification envelope, the model should widen predictive uncertainty rather than remaining overconfident.

The truth data for these studies are generated by a high-resolution one-dimensional two-fluid solver with an entropy-stable flux and with a closure structure richer than the reduced model's parameterization [32]. This deliberate mismatch ensures that the reduced model is not simply

rediscovering its own equations. The solver produces time series of film thickness-like interface variables and of pressure signals at virtual sensor locations, including time-averaged versions to mimic telemetry. From the truth film thickness, Fourier amplitudes are computed on a segment length L and then truncated to a mode set $\mathcal{K}$ representing the reduced state. The reduced model is then identified from a subset of trajectories and evaluated on held-out trajectories with different boundary-condition patterns and with moderate parameter drift in friction scaling.

The first study considers a single annular segment with a slowly varying base state and intermittent wave growth. Boundary conditions are varied to produce episodes of increased gas-core shear, which destabilize intermediate modes and generate wave packets. The reduced model is identified on trajectories spanning a range of mean Reynolds-like conditions. In evaluation, the base state is driven by a boundary change that shifts the operating point outside the identification range for a finite interval, and then returns [33]. Without entropy-bounded closure, a naive truncated amplitude model often exhibits energy pile-up at the highest retained modes during the out-of-range interval, producing negative film thickness in reconstruction with nontrivial frequency. With the entropy-bounded spectral closure, the high-mode damping increases when pile-up indicators activate, and projection corrects rare excursions, resulting in reconstructed film thickness that remains nonnegative throughout the rollout. The fraction of time steps requiring projection remains modest, and when projection is active, the correction is concentrated in high modes, preserving low-mode energy that dominates observable pressure fluctuations.

Spectral prediction is evaluated by comparing the predicted PSD of a differential pressure signal to the truth PSD over windows of interest. The reduced model reproduces the location of dominant peaks associated with convecting wave modes and captures the broadband slope over an intermediate band. Discrepancies appear at very high frequencies, where truth signals include contributions from turbulence and droplet dynamics not represented in the reduced state; in these bands, the model's predicted PSD is lower, but the inferred measurement-noise-plus-unmodeled PSD term compensates, yielding calibrated total uncertainty. Across held-out trajectories, band-limited RMS prediction errors for pressure fluctuations in the primary wave band are reduced relative to a baseline that uses only mean-flow prediction, with improvements typically in the 20% to 40% range depending on sensor filtering and operating condition, while maintaining physically admissible film reconstructions [34].

The second study introduces network coupling. Two annular segments are connected in series, with the downstream segment experiencing a different thermal condition and a different effective friction scaling. Wave packets generated in the upstream segment propagate and influence downstream pressure signatures. The reduced model represents each segment with its own amplitude vector and base state, coupled through boundary conditions at the junction. The coupling is modeled by convecting wave amplitudes across the interface with an attenuation factor and phase shift that depend on mean velocities. In truth, the coupling is more complex because wave reflection and droplet entrainment can occur at the junction, but the reduced coupling is sufficient to test whether the model can propagate spectral energy through a network without instability. The entropy-bounded closure is particularly important here because coupling can amplify certain modes if phase alignment produces constructive interference [35]. In the reduced model, the contractivity regularization on $K(\chi)$ and the spectral dissipation operator prevent such amplification from producing unphysical reconstructions, and predictive uncertainty increases when coupling is uncertain.

In this network study, inference from sparse telemetry is also evaluated. Only boundary pressures and one differential pressure in the upstream segment are assumed measured, while

downstream wave amplitudes are unobserved. The filter estimates upstream amplitudes and propagates them forward to predict downstream spectra. The posterior over downstream amplitudes remains broad, reflecting limited observability, but the predicted downstream pressure PSD remains calibrated in the sense that measured spectra fall within predicted credible bands over most windows. When an additional downstream differential pressure measurement is introduced, posterior uncertainty contracts and predicted spectra sharpen, consistent with the model's measurement transfer structure. This illustrates a key design principle of the framework: it does not assert that all wave modes are inferable from sparse sensors; instead it provides a coherent way to represent what can and cannot be inferred, and it links those limits directly to spectral predictions [36].

The third study probes parameter drift and regime-like changes in effective dynamics. The truth solver is modified so that the effective friction scaling in the liquid film drifts slowly over time, representing surface roughening or deposition. In addition, intermittent changes in entrainment behavior alter effective damping of certain modes, changing the spectral slope. The reduced model does not explicitly model these mechanisms; instead, it represents them through slow drift in base-state parameters and through state-dependent process noise. Identification is performed on trajectories without drift, and evaluation includes drift. The reduced model's mean spectral predictions degrade if parameters are held fixed, but when the base-state drift process is included, the filter can track gradual changes using boundary pressure information, and predictive spectra remain closer to truth. Uncertainty bands widen during periods of entrainment-change events, reflecting that the reduced model cannot deterministically predict such events but can represent increased variability through process noise [37]. This behavior is preferred to an overconfident predictor that would miss rare bursts.

A central concern in reduced modeling is whether the reduced state remains meaningful during strong regime transitions where the annular structure changes qualitatively. The studies include transitions where the film becomes temporarily thicker and wave energy increases, approaching conditions where slugging-like behavior might emerge in a full three-dimensional system. In the one-dimensional truth solver, these appear as strong holdup oscillations and large pressure fluctuations. The reduced model can represent increased wave energy and shifting peaks, but it cannot represent topology changes in interfacial structure beyond the film representation. In these cases, the entropy-bounded closure prevents catastrophic unphysical reconstructions, but the model's predictive uncertainty becomes very large, and the inferred amplitudes become ambiguous. This is an important failure mode: the reduced model is designed to remain stable and honest rather than to overreach [38]. In operational interpretation, large posterior uncertainty and frequent projection activation would be signals that the system has moved outside the reduced model's validity envelope and that additional sensing or a higher-fidelity model is needed.

Uncertainty calibration is evaluated using rank histograms and coverage tests for band-limited energies of pressure signals. For each evaluation window, the model produces an ensemble of predicted PSDs and corresponding RMS values in specified frequency bands. Coverage is assessed by checking whether the truth RMS falls within predicted credible intervals. In bands dominated by retained wave modes, coverage is close to nominal over many windows, indicating that the process noise and measurement noise representations are reasonable. In high-frequency bands dominated by unmodeled effects, coverage depends on the assumed unmodeled PSD term; when this term is included and tuned conservatively, coverage remains acceptable but intervals are wide, reflecting limited predictability of that band.

The computational studies also provide insight into the role of identification regularization [39]. Without contractivity constraints on $K(\chi)$, identified propagators can fit short-horizon

amplitude evolution but become unstable in long rollouts, especially under slight base-state extrapolation. The entropy-bounded projection can prevent unphysical reconstructions, but repeated projection can distort spectral dynamics and reduce predictive fidelity. With contractivity regularization, the propagator remains stable, projection is rarely needed, and spectral predictions remain smoother. This supports the view that admissibility should be enforced primarily through a dissipative reduced dynamics rather than through frequent projection corrections.

Finally, the studies examine sensitivity to truncation choice. Retaining too few modes can underpredict spectral peaks and force process noise to compensate, producing overly diffuse predictions. Retaining too many modes can increase the chance of spectral pile-up and reduce observability, as high modes become weakly constrained by measurements [40]. The entropy-bounded closure reduces the downside of retaining more modes by damping high modes strongly when needed, but observability limits remain. A practical guideline emerging from the studies is to retain modes up to the band where measurement transfer functions have significant gain and where signal-to-noise is not negligible, treating higher bands as unmodeled. This aligns the reduced state dimension with what telemetry can support.

Overall, the computational studies suggest that spectral-entropy reduced modeling can provide a stable and interpretable bridge between annular interfacial wave physics and sparse measurements, yielding predictions of pressure-drop spectra with calibrated uncertainty. The approach does not eliminate the need for high-fidelity modeling in extreme regimes, but it provides a disciplined reduced representation for the variability that most often matters in annular operations.

7. Conclusion

This paper developed a spectral-entropy reduced modeling framework for interfacial wave dynamics in annular gas–liquid two-phase network segments. The framework is built on a separation between slow annular base-state evolution and faster interfacial perturbations represented by a truncated set of Fourier amplitudes [41]. From a two-fluid balance perspective, these amplitudes evolve under linear production and damping terms coupled to base-state variables, with unresolved nonlinear interactions and turbulence represented as stochastic forcing and effective dissipation. A central contribution is an entropy-bounded spectral closure that enforces a discrete energy inequality and introduces a differentiable projection ensuring nonnegative film thickness, bounded void fraction, and physically admissible reconstructions even under long-horizon propagation and sparse-sensor inference updates. Koopman-consistent identification was introduced to learn operating-dependent linear propagators for dominant modes while regularizing for contractivity in a wave-energy norm, reducing instability under extrapolation. Frequency-domain measurement operators were constructed to map wave amplitudes to pressure and differential pressure signals, and a probabilistic inference scheme was described for estimating spectral state and its uncertainty from sparse telemetry including time-averaged measurements.

Computational studies indicated that the entropy-bounded closure mitigates spectral pile-up and prevents unphysical reconstructions, enabling stable predictions of pressure-drop spectra and band-limited RMS fluctuations across operating variations and modest parameter drift. The studies also clarified limitations: when interfacial structure departs strongly from an annular-film representation, the reduced model can remain stable but becomes highly uncertain, and this uncertainty should be interpreted as a validity warning rather than as a precise forecast. The broader implication is that, for annular two-phase networks where co-

herent interfacial waves dominate observable variability, a spectral reduced state with explicit production–dissipation structure and admissibility enforcement can provide fast, uncertainty-aware predictions that are more robust than unconstrained truncations and more operationally focused than purely mean-flow models [42].

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